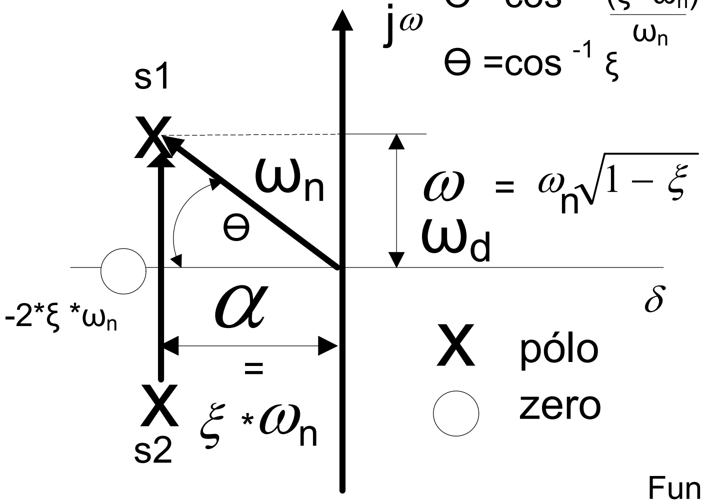
Dedução da Resposta no tempo ao sistema de ordem 2, com pólos complexas conjugados.

$\Theta = \cos^{-1} \dfrac{(\xi * \omega_n)}{\omega_n}$

$\Theta = \cos^{-1} \xi$



Apresentação dos parametros:

α	Parte real do pólo
ω	Parte imaginária do pólo

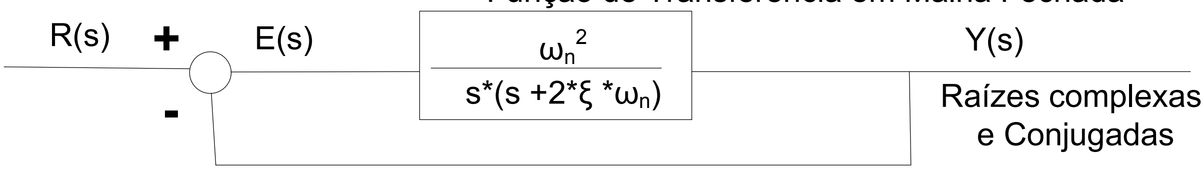
Pólos complexos e conjugados :

$S1 = -\alpha + j\omega$

$S2 = -\alpha - j\omega$

ω_n	Frequencia natural não amortecida
ξ	Coeficiente de amortecimento

Função de Transferência em Malha Fechada



$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2*\xi*\omega_n s + \omega_n^2}$$

$$Y(t) = \frac{(yo) * e^{(-\xi*\omega n*t)}}{\sqrt{1-\xi^2}} * \text{sen}\left(\omega_n\sqrt{1-\xi^2} * t + \Theta\right)$$

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{(s + \xi*\omega_n + j\omega_d) * (s + \xi*\omega_n - j\omega_d)}$$

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{(s + a + jb) * (s + a - jb)}$$

$$Y(s) = \frac{k1}{(s - s1)} + \frac{k2}{(s - s2)}$$

$$k1 = \frac{(yo)\,(s + 2*\xi*\omega_n)}{(s - s1^*)} = \frac{(Y0) * M1 * e^{j\Theta}}{M2 * e^{j¶/2}}$$

$$Y(s) = \frac{k1}{(s - s1)} + \frac{K1^*}{(s - s1^*)}$$

M1 é magnitude de $s + 2*\xi*\omega_n$ M2 é magnitude de $s - s1^*$

$s - s1^* = -\xi*\omega_n + j\omega_d - (-\xi*\omega_n - j\omega_d) = +2*j\omega_d = M2 * e^{j¶/2} = 2 * \omega_d * e^{j¶/2}$

$s + 2*\xi*\omega_n = -\xi*\omega_n + j\omega_d + 2*\xi*\omega_n = -\xi*\omega_n + j\omega_d = M1 * e^{j¶/2} = \omega_n * e^{j\Theta}$

$$k1 = \frac{(Y0) * M1 * e^{j\Theta}}{M2 * e^{j¶/2}} = \frac{(yo)\,\omega_n * e^{j\Theta}}{2 * \omega_n\sqrt{1-\xi^2} * e^{j¶/2}}$$

$$K1 = \frac{(yo) * e^{j(\Theta - ¶/2)}}{2 * \sqrt{1-\xi^2}}$$

$$\omega_d = \omega_n * \sqrt{(1-\xi^2)}$$

$$K2 = K1^*$$

$$K2 = \frac{(yo) * e^{j(¶/2 - \Theta)}}{2 * \sqrt{1-\xi^2}}$$

$$K1 = \frac{(yo) * e^{j(\Theta - ¶/2)}}{2* \sqrt{1-\xi^2}}$$

$$K2 = \frac{(yo) * e^{j(¶/2 - \Theta)}}{2* \sqrt{1-\xi^2}}$$

$$Y(t) = K1 * e^{s1 *t} + K2 * e^{s2 *t}$$

$$Y(t) = \frac{(yo) * e^{j(\Theta - ¶/2)}}{2* \sqrt{1-\xi^2}} * e^{s1 *t} + \frac{(yo) * e^{j(¶/2 - \Theta)}}{2* \sqrt{1-\xi^2}} * e^{s2 *t}$$

$$S1 = -\xi *\omega_n + j\omega_n * \sqrt{1-\xi^2} \qquad S1 = -\xi *\omega_n + j\omega_n * B$$

$$S2 = -\xi *\omega_n - j\omega_n * \sqrt{1-\xi^2} \qquad S2 = -\xi *\omega_n - j\omega_n * B$$

$$B = \sqrt{1-\xi^2}$$

$$e^{s1 *t} = e^{(-\xi *\omega n + j_{\omega n B} *t)}$$

$$e^{s2 *t} = e^{(-\xi *\omega n - j_{\omega n B} *t)}$$

$$Y(t) = \frac{(yo) * e^{j(\Theta - ¶/2)}}{2* \sqrt{1-\xi^2}} * e^{(-\xi *\omega n + j_{\omega n B} *t)} + \frac{(yo) * e^{j(¶/2 - \Theta)}}{2* \sqrt{1-\xi^2}} * e^{(-\xi *\omega n - j_{\omega n B} *t)}$$

$$Y(t) = \frac{(yo) * e^{j(\Theta - ¶/2)}}{2* \sqrt{1-\xi^2}} * e^{(-\xi *\omega n*t)} e^{(j_{\omega n B} *t)} + \frac{(yo) * e^{j(¶/2 - \Theta)}}{2* \sqrt{1-\xi^2}} * e^{(-\xi *\omega n*t)} e^{(-j_{\omega n B} *t)}$$

$$Y(t) = \frac{(yo) * e^{(-\xi *\omega n*t)}}{2* \sqrt{1-\xi^2}} * \left(e^{j(\Theta - ¶/2)} * e^{j_{\omega n B} *t} + e^{j(¶/2 - \Theta)} * e^{-j_{\omega n B} *t}\right)$$

$$Y(t) = \frac{(yo) * e^{(-\xi *\omega n*t)}}{2* \sqrt{1-\xi^2}} * \left(e^{j(\Theta - ¶/2)} * e^{j_{\omega n B} *t} + e^{-j(\Theta - ¶/2)} * e^{-j_{\omega n B} *t}\right)$$

$$Y(t) = \frac{(yo) * e^{(-\xi *\omega n*t)}}{2* \sqrt{1-\xi^2}} * \left(e^{j(\Theta - ¶/2 + \omega n B *t)} + e^{-j(\Theta - ¶/2 + \omega n B *t)}\right)$$

Identidade de Euler : $e^{jx} = \cos x + j \operatorname{sen} x$

$$\begin{aligned} & e^{jx} = \cos x + j \operatorname{sen} x \\ + \; & e^{-jx} = \cos x - j \operatorname{sen} x \\ \hline & e^{jx} + e^{-jx} = 2 * \cos x \end{aligned}$$

$$\left(e^{j(\Theta - ¶/2 + \omega n*B *t)} + e^{-j(\Theta - ¶/2 + \omega n*B *t)}\right) = 2 * \cos(\Theta - ¶/2 + \omega_n *B *t)$$

$$\left(e^{j(\Theta - ¶/2 + B *t)} + e^{-j(\Theta - ¶/2 + B *t)}\right) = 2 *\cos(\Theta - ¶/2 + \omega_n B *t) = 2 *\cos(B *t + \Theta - ¶/2)$$

$$Y(t) = \frac{(yo) * e^{(-\xi *\omega n*t)}}{2* \sqrt{1-\xi^2}} * 2 *\cos(\omega_n *B *t + \Theta - ¶/2)$$

$$\cos(x - ¶/2) = \operatorname{sen} x$$

$$Y(t) = \frac{(yo) * e^{(-\xi *\omega n*t)}}{2* \sqrt{1-\xi^2}} * 2 * \operatorname{sen}(\omega_n \sqrt{1-\xi^2} *t + \Theta)$$

$$Y(t) = \frac{(yo) * e^{(-\xi *\omega n*t)}}{\sqrt{1-\xi^2}} * \operatorname{sen}(\omega_n \sqrt{1-\xi^2} *t + \Theta)$$