

Tarefa 2

Para o sistema dinâmico abaixo, encontre K e Km necessários para que o sobressinal seja, ao menos, menor que 5% e o tempo de assentamento (2%) menor que 5 segundos. Utilize Matlab/Simulink para verificar os resultados

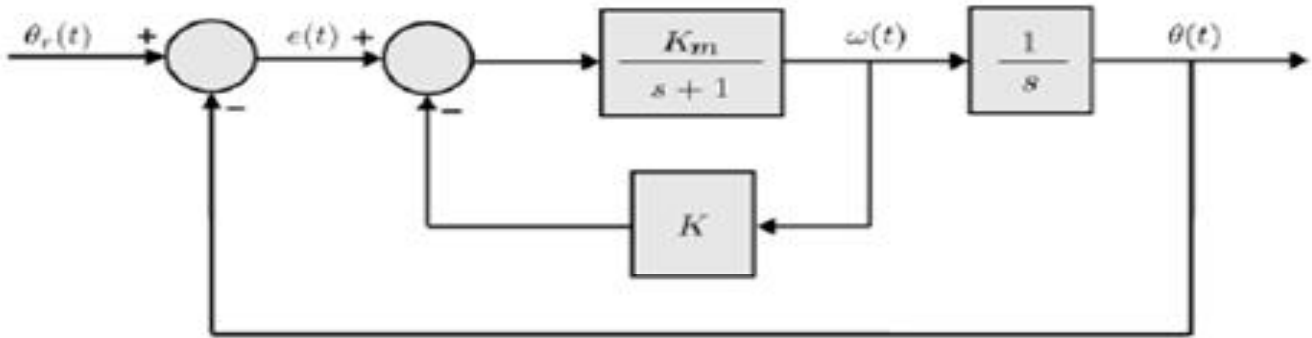


Figura 3.31 Servossistema com realimentação tacométrica.

$$G(s) \cdot H(s) = \frac{K_m}{s+1}$$

$$H(s) = K$$

Máximo Pico Percentual MP%

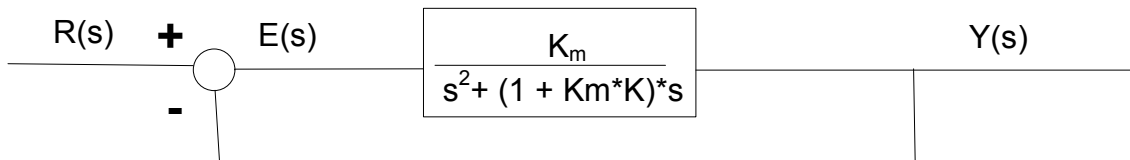
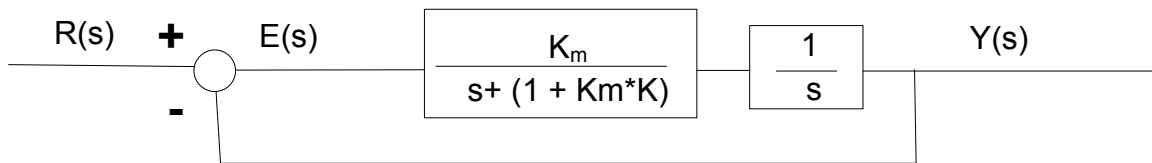
$$MP = 100 \cdot e^{-\left(\xi \sqrt{1-\xi^2}\right) \cdot \pi}$$

$$MP(\%) < 5\%$$

$$T_s < 5 \text{ segundos}$$

Tempo de estabelecimento: t_s

$$t_s = 4 \cdot T = \frac{4}{\xi \cdot \omega_n} = \frac{4}{\sigma} \quad \text{Critério de 2\%}$$



$$G(s) = \frac{K_m}{s^2 + (1 + K_m \cdot K) \cdot s + K_m}$$

Máximo Pico Percentual MP%

$$MP = 100 e^{-(\xi / \sqrt{1-\xi^2}) * \pi}$$

MP(%) < 5%

$$\ln(e^{-(\xi / \sqrt{1-\xi^2}) * \pi}) < \ln(MP \%)$$

$$A = | \ln(MP\%) |$$

$$-(\xi / \sqrt{1-\xi^2}) * \pi < -A$$

$$\xi * \pi > A * \sqrt{1-\xi^2}$$

$$\xi^2 * \pi^2 / A^2 > 1 - \xi^2$$

$$\xi^2 (\pi^2 / A^2 + 1) > 1$$

$$\xi^2 > \frac{1}{(\pi^2 / A^2 + 1)}$$

$$\xi > \sqrt{\frac{1}{(\pi^2 / A^2 + 1)}}$$

Exemplo:

MP% = 5% = 0,05

A = 2,9957 = | ln(0,05) |

$\pi^2 / A^2 + 1 = 2,1039$ $1 / \sqrt{(\pi^2 / A^2 + 1)} =$

$\xi > 0,6894$

Tempo de acomodação : ts

$$t_s = 4 * T = \frac{4}{\xi * \omega_n} = \frac{4}{\sigma} \quad \text{Critério de 2\%}$$

ts < 5 segundos

$\xi > 0,6901$

$$\frac{4}{\sigma} < t_s = 5 \text{ segundos}$$

$$\sigma > \frac{4}{t_s} \quad \xi * \omega_n > \frac{4}{t_s}$$

$$\sigma > \frac{4}{5} \quad \omega_n > \frac{4}{5 * \xi}$$

Exemplo:

$\xi > 0,6901$

$$\omega_n > \frac{4}{5 * \xi}$$

$$\omega_n > \frac{4}{5 * 0,7}$$

$$\omega_n > 1,1592$$

$$\omega_n > \frac{4}{5 * 0,7}$$

$\omega_n = 1,1592$

Valor adotado

$$\frac{Y(s)}{R(s)} = \frac{K_m}{s^2 + (1 + K_m * K) * s + K_m}$$

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2 * \xi * \omega_n * s + \omega_n^2}$$

Calculo do valor da constante Km:

Usando $\omega_n = 1,1592$ $K_m = \omega_n^2$

$K_m = 1,1592^2$

$K_m = 1,3438$

$\xi = 0,6901$

$\omega_n^2 = 1,3438$

Calculo do valor da constante K:

$$K = (2 * \xi * \omega_n - 1) / K_m$$

$$2 * \xi * \omega_n = (1 + K_m * K) =$$

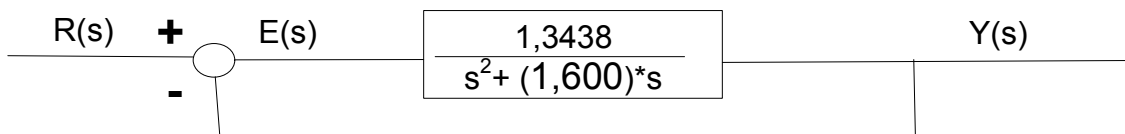
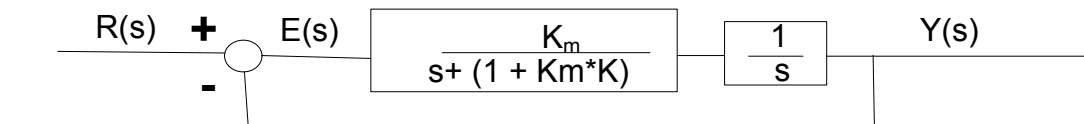
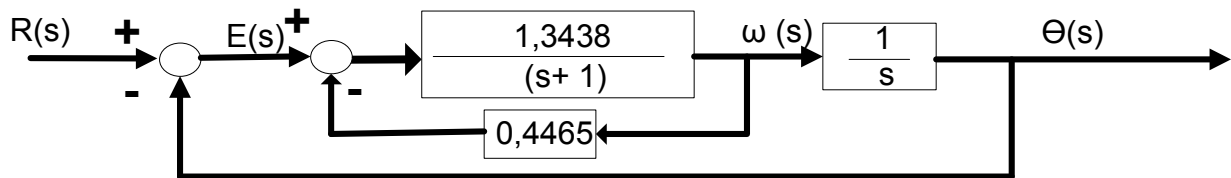
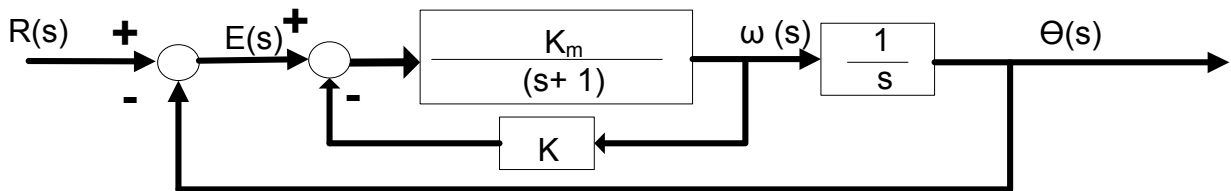
$$2 * \xi * \omega_n = 2 * 0,7 * 1,14 = 1,600$$

$$K = (2 * 1,1592 * 1,3438 - 1) / 1,3438$$

K = 0,4465

Resumo: (1) $\xi > \sqrt{\frac{1}{(\pi^2 / A^2 + 1)}}$ (2) $\omega_n > \frac{4}{ts \cdot \xi}$ (3) $K_m = \omega_n^2$
 $A = |\ln(MP\%)|$ (4) $K = (2 \cdot \xi \cdot \omega_n - 1) / K_m$ (5) $2 \cdot \xi \cdot \omega_n = (1 + K_m \cdot K) =$

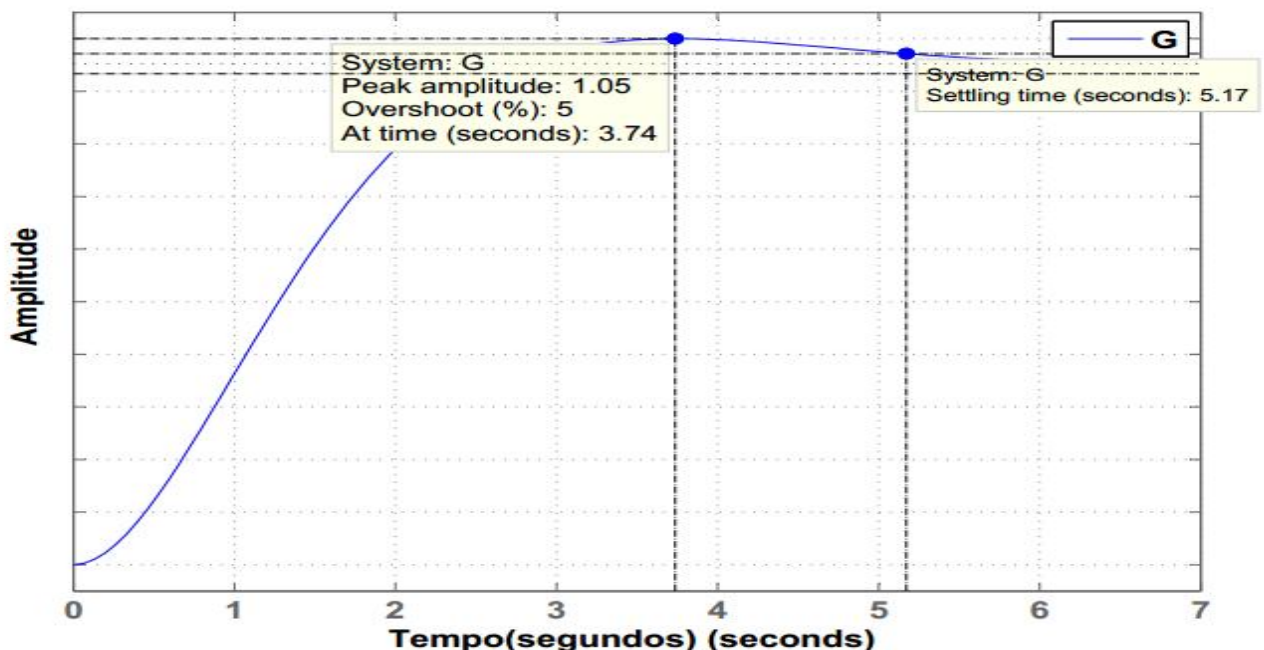
Exemplo: $MP\% < 5\%$ $ts < 5s$ critério 2%
 $\xi = 0,6901$ $\omega_n = 1,1592$ $K_m = 1,3438$ $K = 0,4465$ $2 \cdot \xi \cdot \omega_n = 1,600$



$$\frac{Y(s)}{R(s)} = \frac{1,3438}{s^2 + (1,600)s + 1,3438}$$

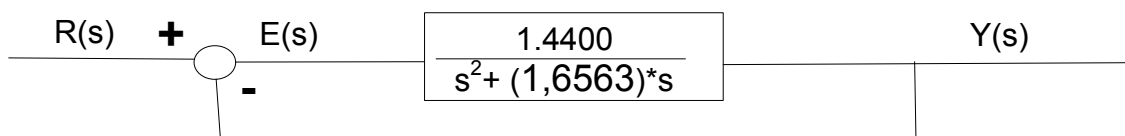
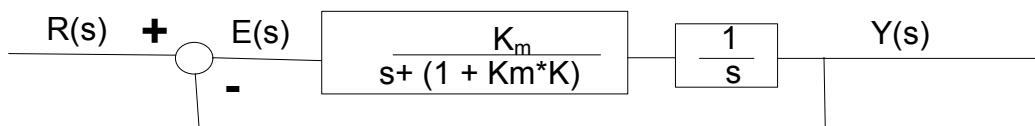
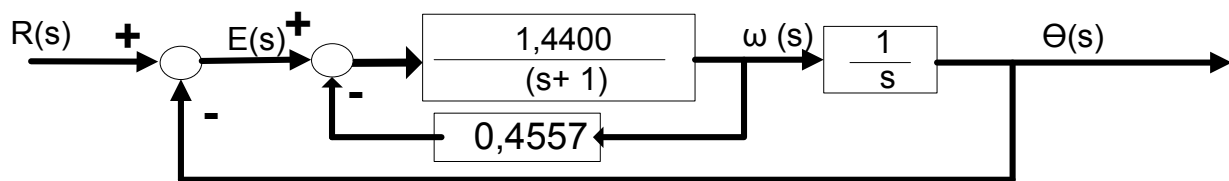
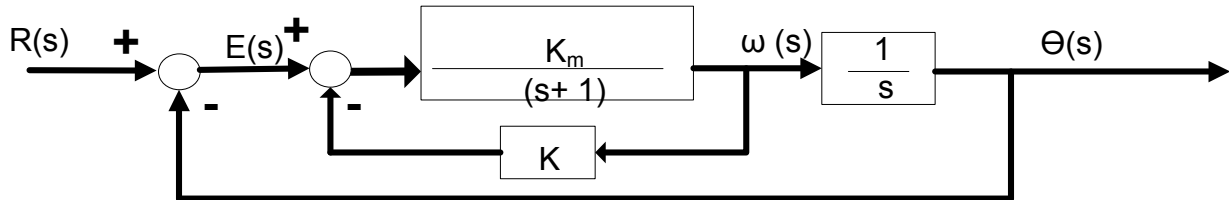
$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2 \cdot \xi \cdot \omega_n s + \omega_n^2}$$

Step Response - Resposta ao Degrau $G(s) = 1.3438 / (s^2 + 1.600s + 1.3438)$



Resumo: (1) $\xi > \sqrt{\frac{1}{(\pi^2 / A^2 + 1)}}$ (2) $\omega_n > \frac{4}{ts \cdot \xi}$ (3) $K_m = \omega_n^2$
 $A = |\ln(MP\%)|$ (4) $K = (2 \cdot \xi \cdot \omega_n - 1) / K_m$ (5) $2 \cdot \xi \cdot \omega_n = (1 + K_m \cdot K) =$

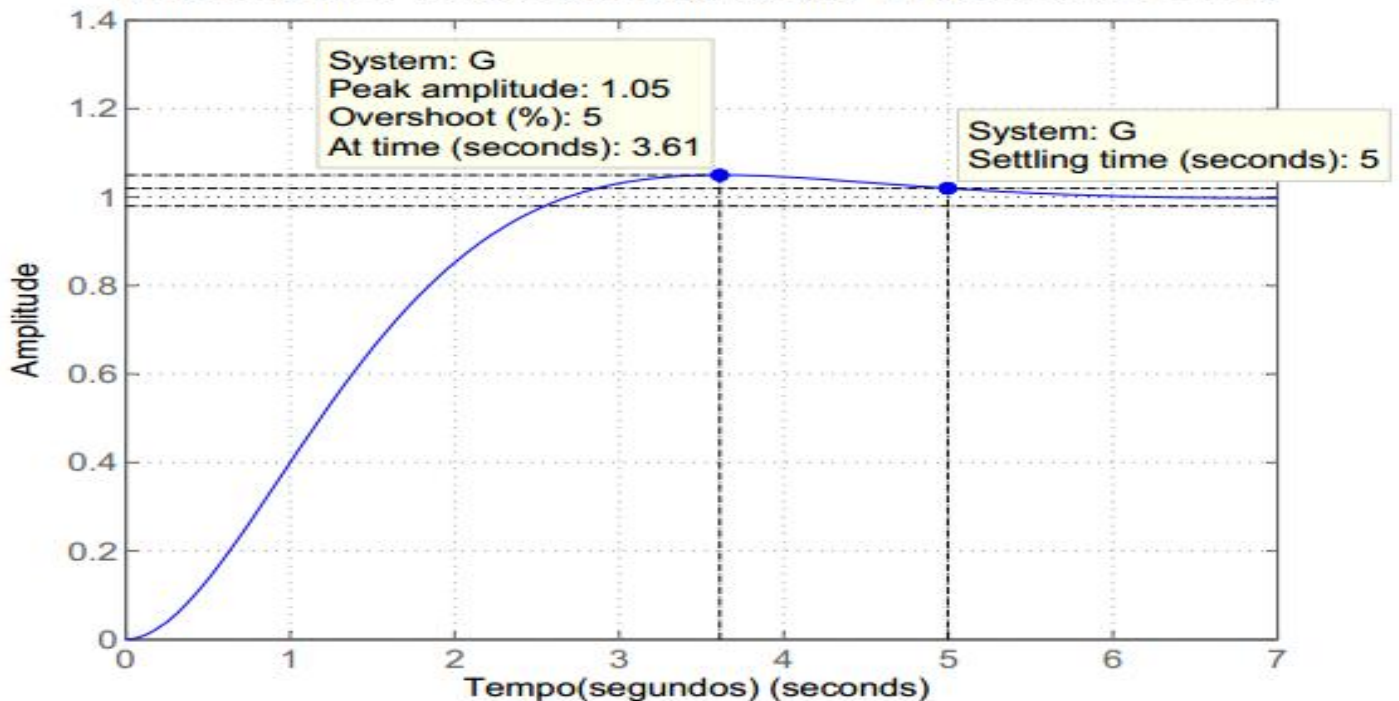
Exemplo: $MP\% < 5\%$ $ts < 5s$ critério 2%
 $\xi = 0,6901$ $\omega_n = 1.2000$ $K_m = 1,4400$ $K = 0,4557$ $2 \cdot \xi \cdot \omega_n = 1,6563$



$$\frac{Y(s)}{R(s)} = \frac{1.4400}{s^2 + (1,6563)s + 1,3438}$$

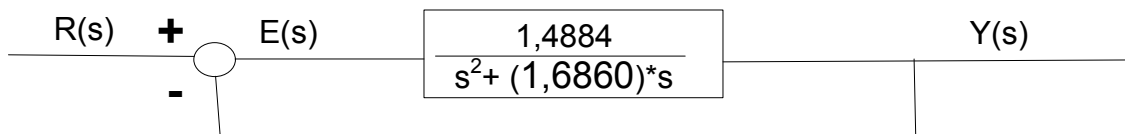
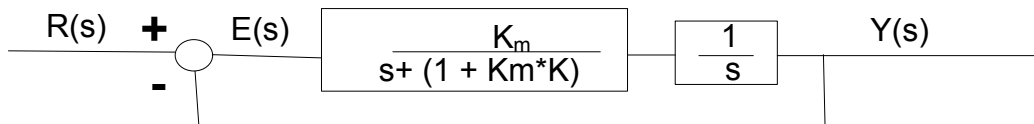
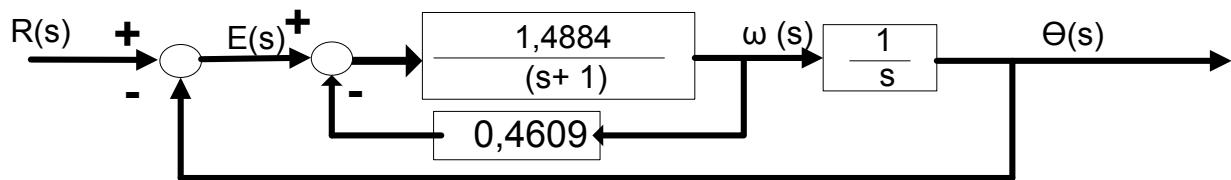
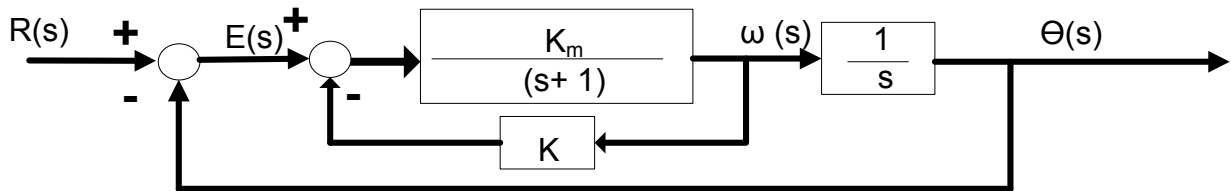
$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2 \cdot \xi \cdot \omega_n s + \omega_n^2}$$

Step Response - Resposta ao Degrau $G(s) = 1.44 / (s^2 + 1.6563s + 1.44)$



Resumo: (1) $\xi > \sqrt{\frac{1}{(\pi^2 / A^2 + 1)}}$ (2) $\omega_n > \frac{4}{ts \cdot \xi}$ (3) $K_m = \omega_n^2$
 $A = |\ln(MP\%)|$ (4) $K = (2 \cdot \xi \cdot \omega_n - 1) / K_m$ (5) $2 \cdot \xi \cdot \omega_n = (1 + K_m \cdot K) =$

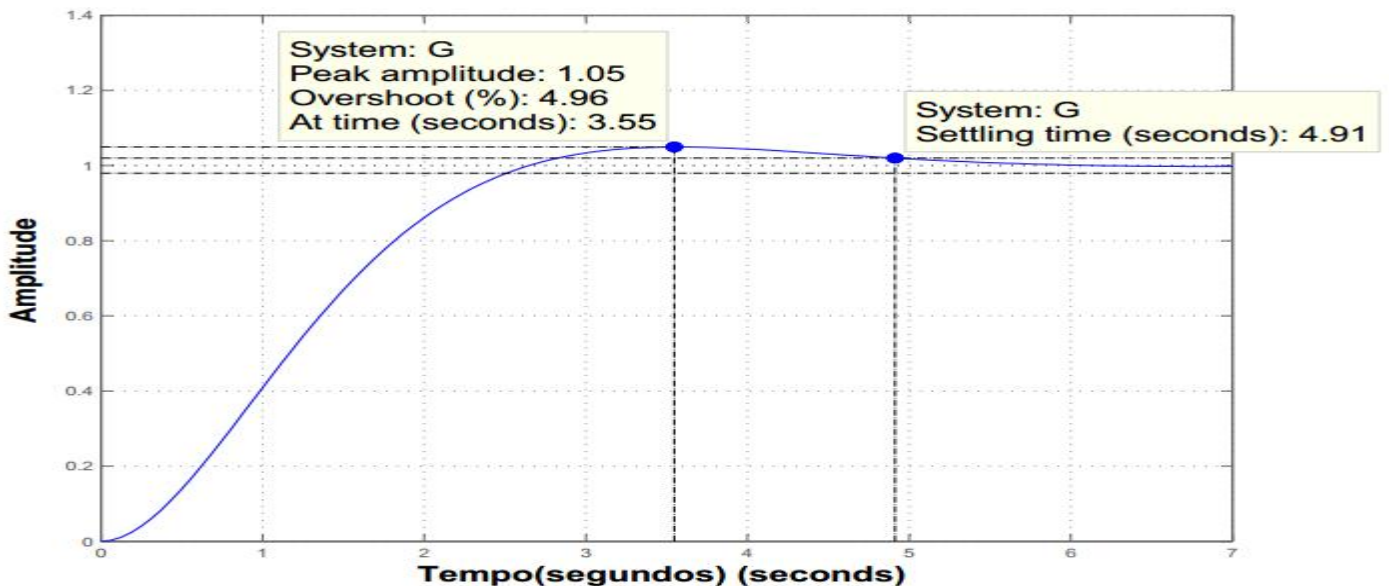
Exemplo: MP% < 5% $ts < 5s$ critério 2%
 $\xi = 0.691$ $\omega_n = 1.22$ $K_m = 1,4884$ $K = 0,4609$ $2 \cdot \xi \cdot \omega_n = 1,6860$

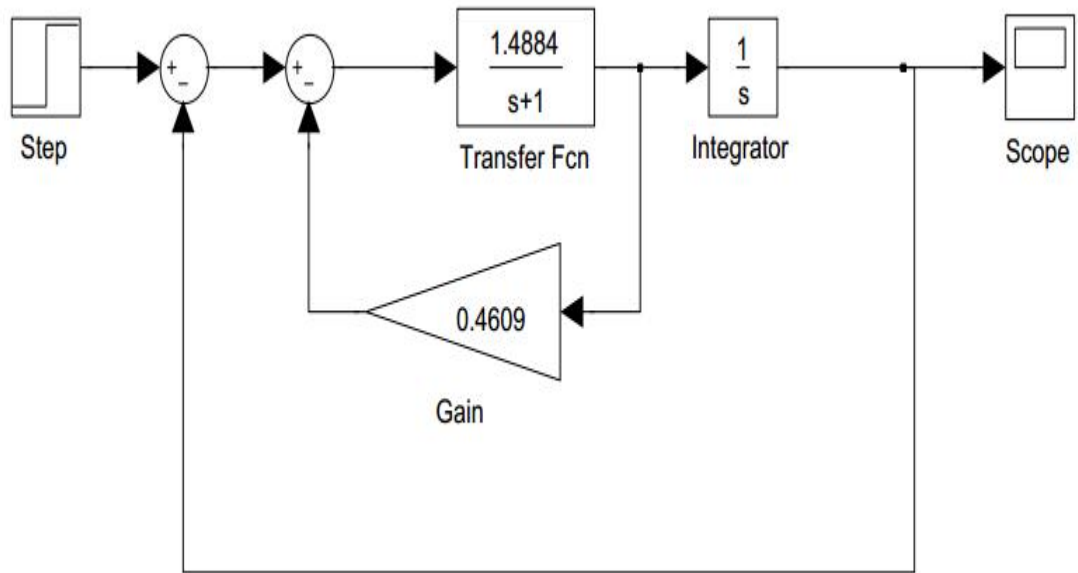


$$\frac{Y(s)}{R(s)} = \frac{1,4884}{s^2 + (1,6860) \cdot s + 1,4884}$$

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2 \cdot \xi \cdot \omega_n \cdot s + \omega_n^2}$$

Step Response - Resposta ao Degrau $G(s) = 1.4884 / (s^2 + 1.6860 \cdot s + 1.4884)$





Resposta ao Degrau $G(s)=\frac{1.4884}{s^2+1.6860s+1.4884}$

