

Tarefa 3

Para o sistema dinâmico abaixo, encontre os polos da malha fechada e o erro em regime permanente para: (Utilize Matlab/Simulink para verificar os resultados)

- $r(t)$ = degrau unitário e $d(t) = 0$
- $d(t)$ = degrau unitário e $r(t) = 0$
- $r(t)$ = rampa unitária e $d(t) = 0$
- $d(t)$ = rampa unitária e $r(t) = 0$
- Com amplificador $C(s) = 10/s$ refaça os itens (a), (b), (c) e (d)

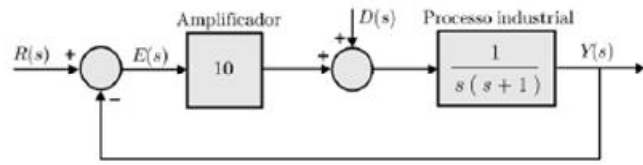
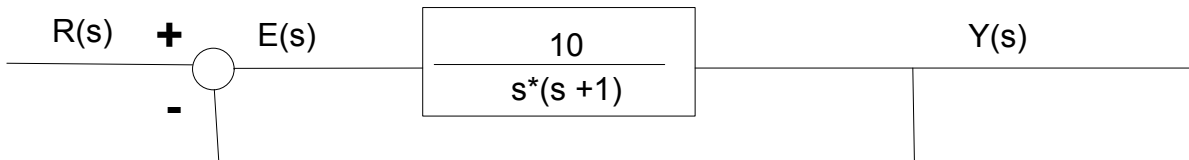
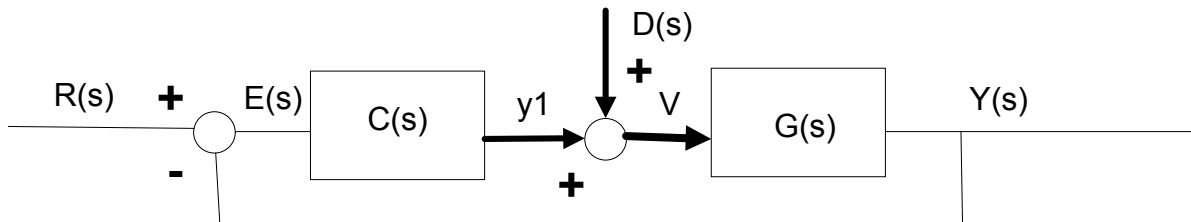


Figura 3.47 Processo industrial com entradas de referência $R(s)$ e de perturbação $D(s)$.



$$\frac{Y(s)}{R(s)} = \frac{C(s) \cdot G(s)}{1 + G(s) \cdot C(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{10}{s^2 + s + 10} \quad D(s) = 0$$



(1) $Y(s) = G(s) \cdot V$

(2) $E(s) = -Y(s)$

(3) $Y_1(s) = C(s) \cdot E(s)$

(4) $Y_1(s) = C(s) \cdot (-Y(s))$

(5) $V(s) = D(s) + Y_1(s)$

(6) $V(s) = D(s) - C(s) \cdot Y(s)$

(7) $Y(s) = G(s) \cdot [D(s) - C(s) \cdot Y(s)]$

(8) $Y(s) + Y(s) \cdot G(s) \cdot C(s) = G(s) \cdot D(s)$

(9) $Y(s) (1 + G(s) \cdot C(s)) = G(s) \cdot D(s)$

$$\frac{Y(s)}{D(s)} = \frac{G(s)}{1 + G(s) \cdot C(s)} \quad R(s) = 0$$

$$Y(s) = \frac{C(s) \cdot G(s)}{1 + G(s) \cdot C(s)} \cdot R(s) + \frac{G(s)}{1 + G(s) \cdot C(s)} \cdot D(s)$$

$$Y(s) = \frac{C(s) \cdot G(s)}{1 + G(s) \cdot C(s)} \cdot R(s) \quad D(s) = 0$$

$$E(s) = \frac{R(s) \cdot (1 + G(s) \cdot C(s)) - C(s) \cdot G(s) \cdot R(s)}{1 + G(s) \cdot C(s)}$$

$$E(s) = R(s) - Y(s) \quad D(s) = 0$$

$$E(s) = \frac{R(s)}{1 + G(s) \cdot C(s)}$$

$$E(s) = R(s) - \frac{C(s) \cdot G(s)}{1 + G(s) \cdot C(s)} \cdot R(s)$$

$$E_{ss} = \lim_{s \rightarrow 0} \frac{s \cdot R(s)}{1 + G(s) \cdot C(s)} \quad D(s) = 0$$

$$E(s) = -Y(s) \quad R(s) = 0$$

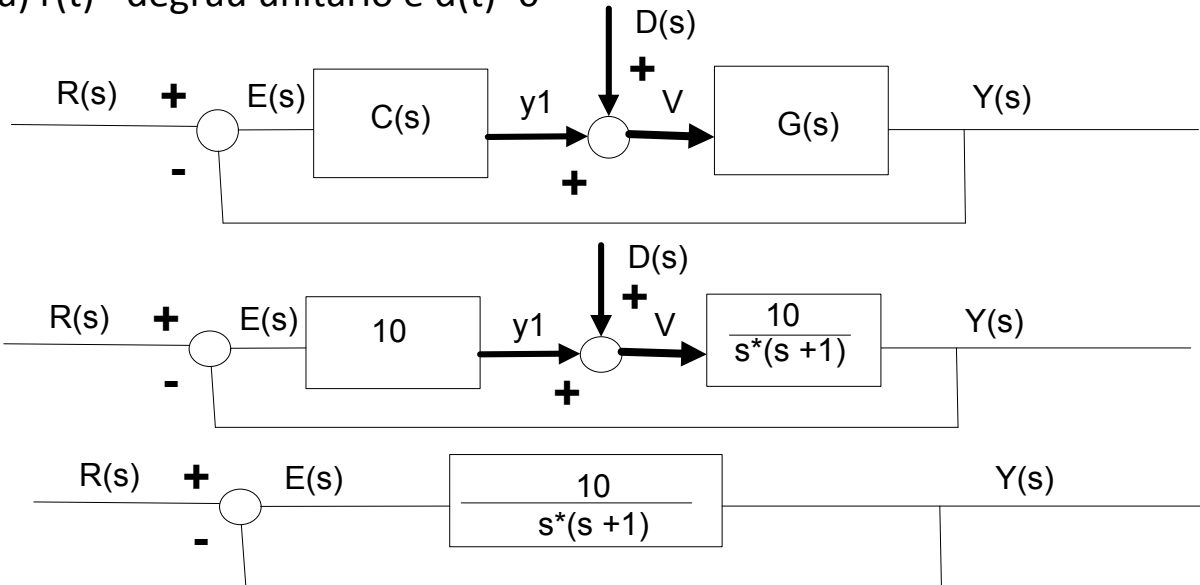
$$E(s) = \frac{-G(s)}{1 + G(s) \cdot C(s)} \cdot D(s)$$

$$E_{ss} = \lim_{s \rightarrow 0} \frac{-s \cdot G(s)}{1 + G(s) \cdot C(s)} \cdot D(s) \quad R(s) = 0$$

Item(a)

$$Y(s) = \frac{C(s)*G(s)}{1 + G(s)*C(s)} *R(s) + \frac{G(s)}{1 + G(s)*C(s)} *D(s)$$

a) $r(t)$ = degrau unitário e $d(t)$ =0



$$\frac{Y(s)}{R(s)} = \frac{C(s)*G(s)}{1 + G(s)*C(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{10}{s^2 + s + 10} \quad \begin{matrix} D(s) = 0 \\ R(s) = 1/s \end{matrix}$$

$$Ess = \lim_{s \rightarrow 0} \frac{s * R(s)}{1 + G(s)*C(s)} \quad D(s) = 0$$

$$A = \lim_{s \rightarrow 0} [1 + G(s)*C(s)]$$

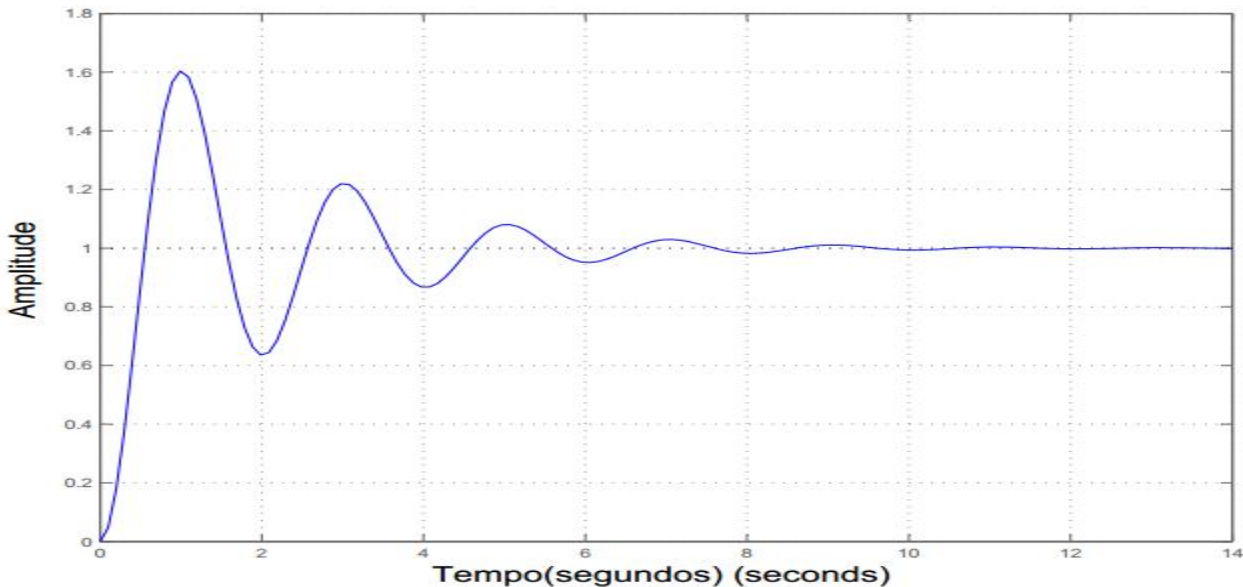
$$Ess = \lim_{s \rightarrow 0} \frac{s}{[1 + G(s)*C(s)] * s} \quad D(s) = 0$$

$$A = \lim_{s \rightarrow 0} 1 + \frac{10}{s*(s+1)} = \infty$$

$$Ess = 0$$

$$Ess = \lim_{s \rightarrow 0} \frac{1}{\lim_{s \rightarrow 0} [1 + G(s)*C(s)]} = \frac{1}{A}$$

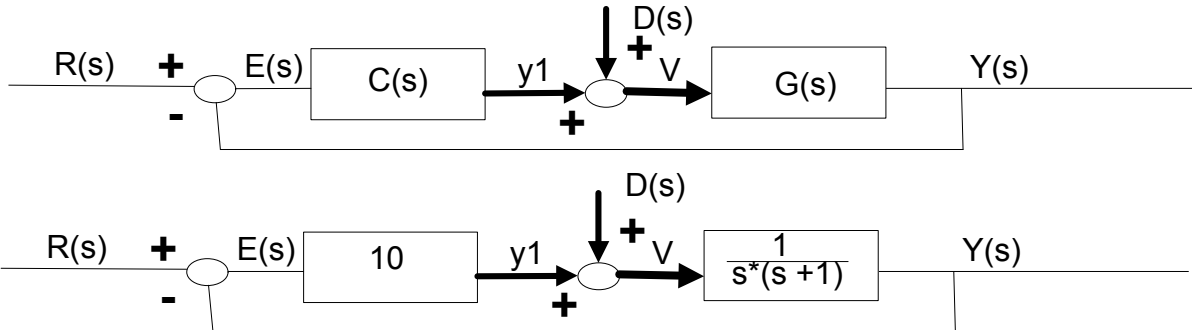
Step Response - Resposta ao Degrau $M(s)=10/(s^2+1*s+10)$



Item(b)

$$Y(s) = \frac{C(s)*G(s)}{1 + G(s)*C(s)} *R(s) + \frac{G(s)}{1 + G(s)*C(s)} *D(s)$$

b) d(t)= degrau unitário e r(t)=0



$$\frac{Y(s)}{D(s)} = \frac{G(s)}{1 + G(s)*C(s)}$$

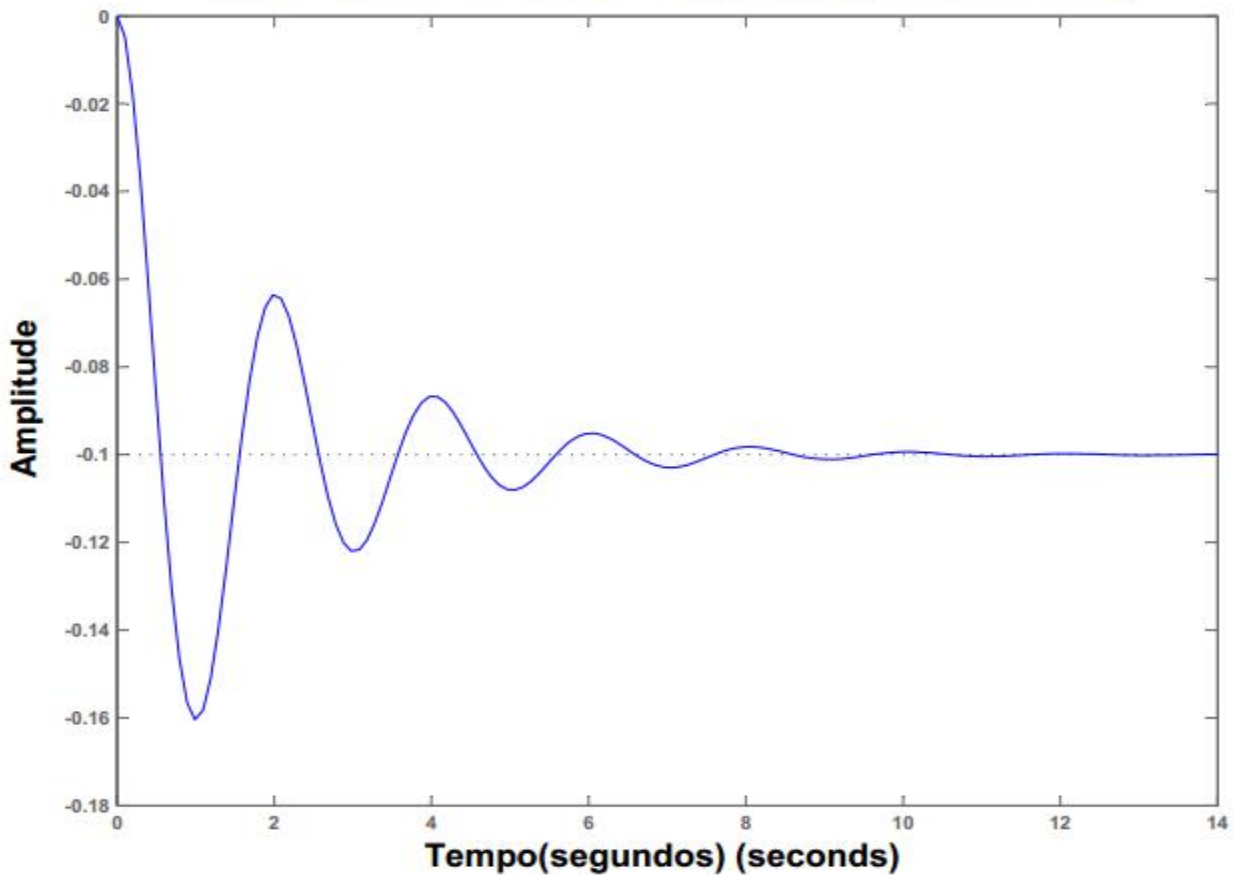
$$\frac{Y(s)}{D(s)} = \frac{1}{s^2 + s + 10} \quad \begin{matrix} D(s) = 1/s \\ R(s) = 0 \end{matrix}$$

$$Ess = \lim_{s \rightarrow 0} \frac{-s * G(s)}{1 + G(s)*C(s)} * D(s) \quad R(s) = 0$$

$$Ess = \lim_{s \rightarrow 0} \frac{-s * 1}{s^2 + s + 10} * \frac{1}{s}$$

$$Ess = - 1/10$$

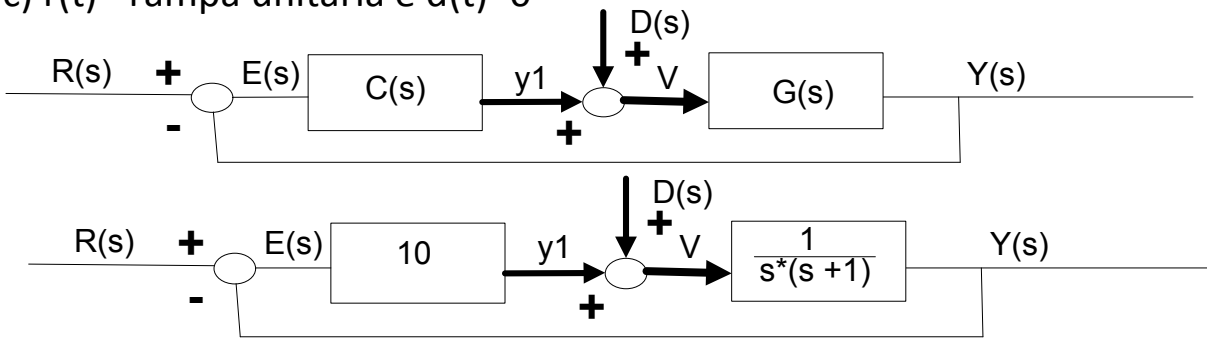
Step Response - Resposta ao Degrau $M(s) = -1/(s^2 + 1*s + 10)$



Item(c)

$$Y(s) = \frac{C(s)*G(s)}{1 + G(s)*C(s)} *R(s) + \frac{G(s)}{1 + G(s)*C(s)} *D(s)$$

c) r(t)= rampa unitária e d(t)=0



$$\frac{Y(s)}{R(s)} = \frac{C(s)*G(s)}{1 + G(s)*C(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{10}{s^2 + s + 10} \quad \begin{matrix} D(s) = 0 \\ R(s) = 1/s^2 \end{matrix}$$

$$Ess = \lim_{s \rightarrow 0} \frac{s * R(s)}{1 + G(s)*C(s)} \quad D(s) = 0$$

$$A = \lim_{s \rightarrow 0} [1 + G(s)*C(s)]$$

$$Ess = \lim_{s \rightarrow 0} \frac{s}{[1 + G(s)*C(s)] * s^2} \quad D(s) = 0$$

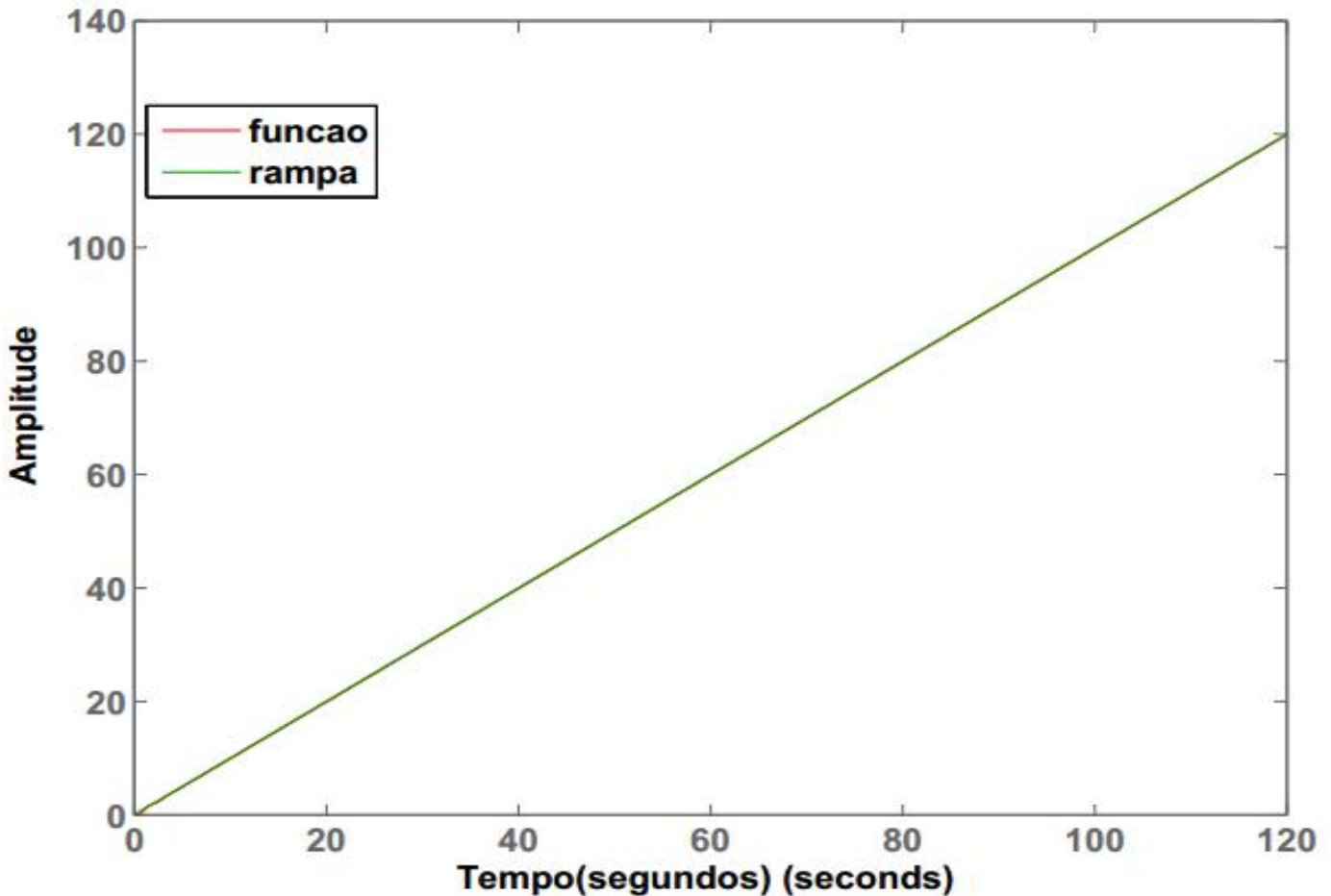
$$A = \lim_{s \rightarrow 0} \left[1 + \frac{10}{s*(s+1)} * s \right] = 11$$

$$Ess = \lim_{s \rightarrow 0} \frac{1}{\lim_{s \rightarrow 0} [1 + G(s)*C(s)] * s} = \frac{1}{A}$$

$$\lim_{s \rightarrow 0} \frac{10}{s*(s+1)} * s$$

$$Ess = 1/11$$

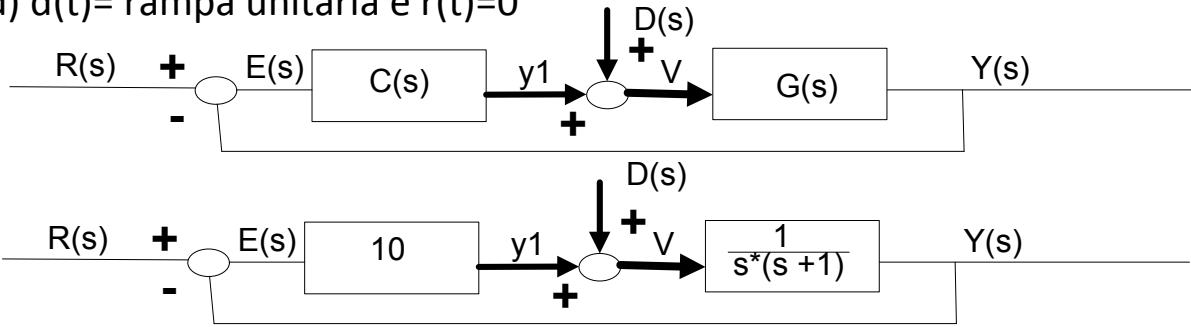
Resposta a Rampa M(s)=10/(s²+1*s+10)



Item(d)

$$Y(s) = \frac{C(s)*G(s)}{1 + G(s)*C(s)} *R(s) + \frac{G(s)}{1 + G(s)*C(s)} *D(s)$$

d) d(t)= rampa unitária e r(t)=0



$$\frac{Y(s)}{D(s)} = \frac{G(s)}{1 + G(s)*C(s)}$$

$$\frac{Y(s)}{D(s)} = \frac{1}{s^2 + s + 10} \quad \begin{matrix} D(s) = 1/s^2 \\ R(s) = 0 \end{matrix}$$

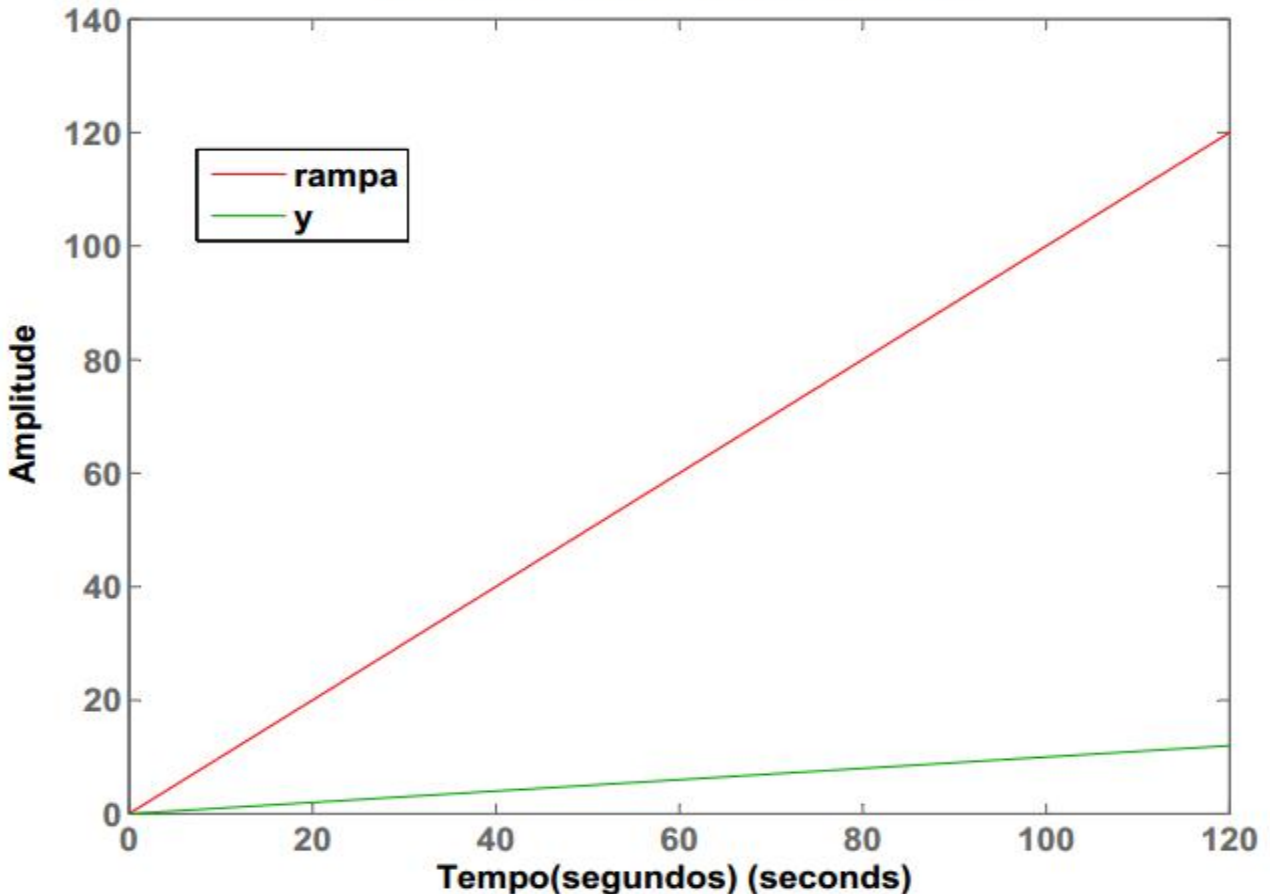
$$Ess = \lim_{s \rightarrow 0} \frac{-s * G(s)}{1 + G(s)*C(s)} * D(s) \quad R(s) = 0$$

$$Ess = \lim_{s \rightarrow 0} \frac{-s * 1}{s^2 + s + 10} * \frac{1}{s^2}$$

$$Ess = \lim_{s \rightarrow 0} \frac{-1}{s^2 + s + 10} * \frac{1}{s}$$

$$Ess = -\infty$$

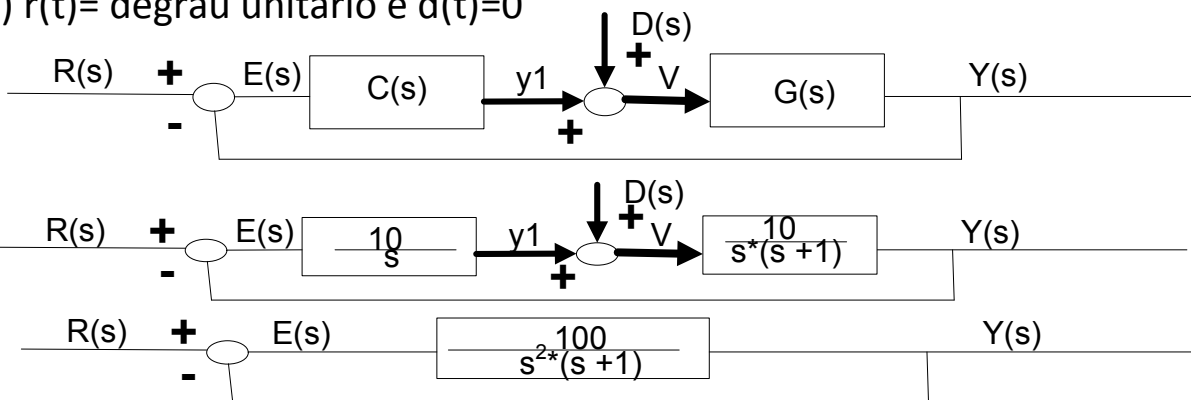
Resposta a Rampa $M(s)=1/(s^2+s+10)$



Item(a) Refazer para $C(s) = 10/s$

$$Y(s) = \frac{C(s)*G(s)}{1 + G(s)*C(s)} *R(s) + \frac{G(s)}{1 + G(s)*C(s)} *D(s)$$

a) $r(t)$ = degrau unitário e $d(t)=0$



$$\frac{Y(s)}{R(s)} = \frac{C(s)*G(s)}{1 + G(s)*C(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{100}{s^3 + s^2 + 100} \quad \begin{matrix} D(s) = 0 \\ R(s) = 1/s \end{matrix}$$

$$Ess = \lim_{s \rightarrow 0} \frac{s * R(s)}{1 + G(s)*C(s)} \quad D(s) = 0$$

$$A = \lim_{s \rightarrow 0} [1 + G(s)*C(s)]$$

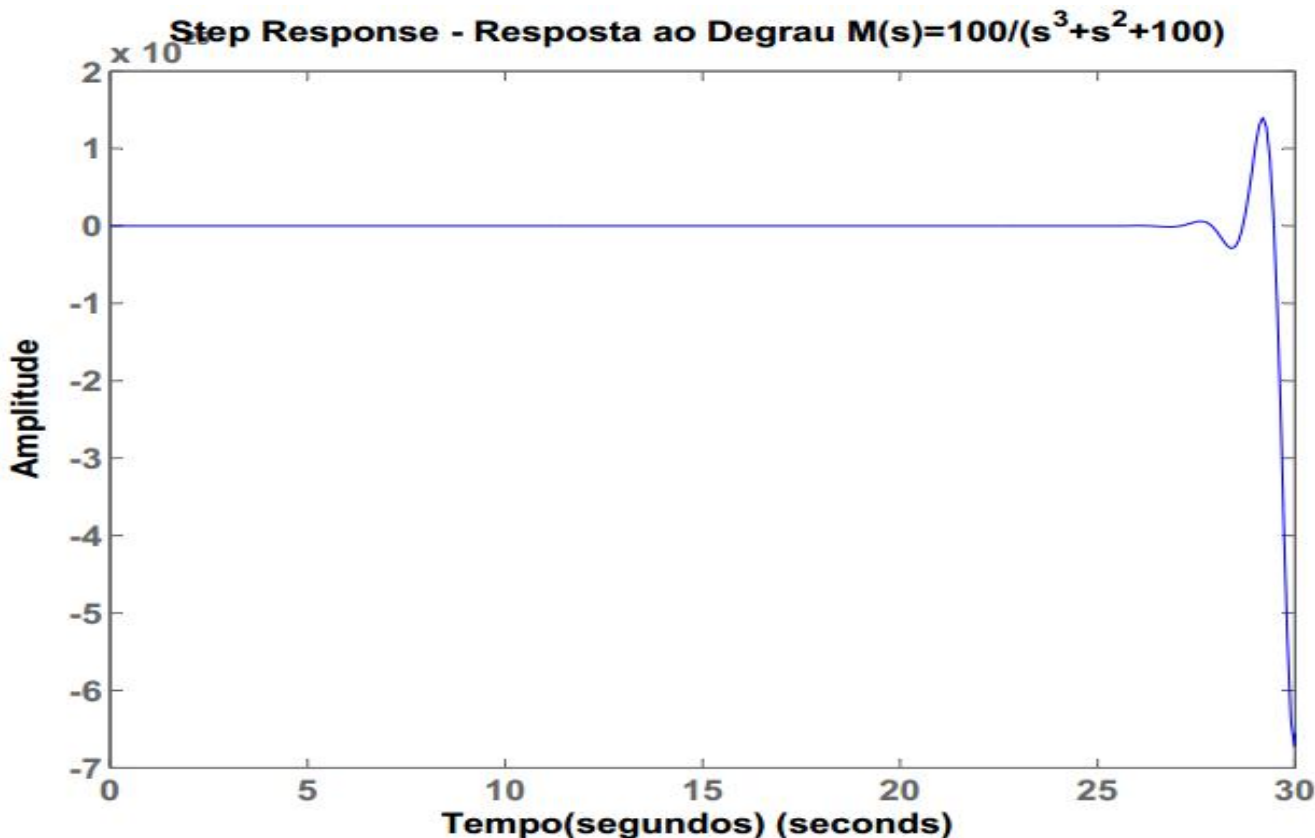
$$Ess = \lim_{s \rightarrow 0} \frac{s}{[1 + G(s)*C(s)] * s} \quad D(s) = 0$$

$$A = \lim_{s \rightarrow 0} 1 + \frac{100}{s^2*(s+1)} = \infty$$

$$Ess = 0$$

$$Ess = \lim_{s \rightarrow 0} \frac{1}{\lim_{s \rightarrow 0} [1 + G(s)*C(s)]} = \frac{1}{A}$$

Análise do erro inválida pois Função de transferência tem pólos no semi-plano direito , resposta instável

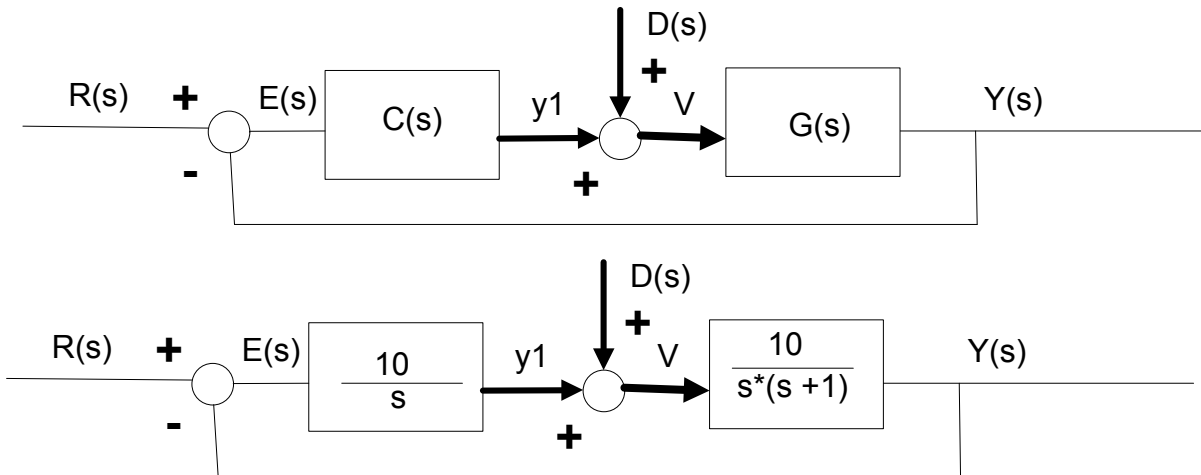


Item(b)

Refazer para $C(s) = 10/s$

$$Y(s) = \frac{C(s) \cdot G(s)}{1 + G(s) \cdot C(s)} \cdot R(s) + \frac{G(s)}{1 + G(s) \cdot C(s)} \cdot D(s)$$

b) $d(t)$ = degrau unitário e $r(t)=0$



$$\frac{Y(s)}{D(s)} = \frac{G(s)}{1 + G(s) \cdot C(s)}$$

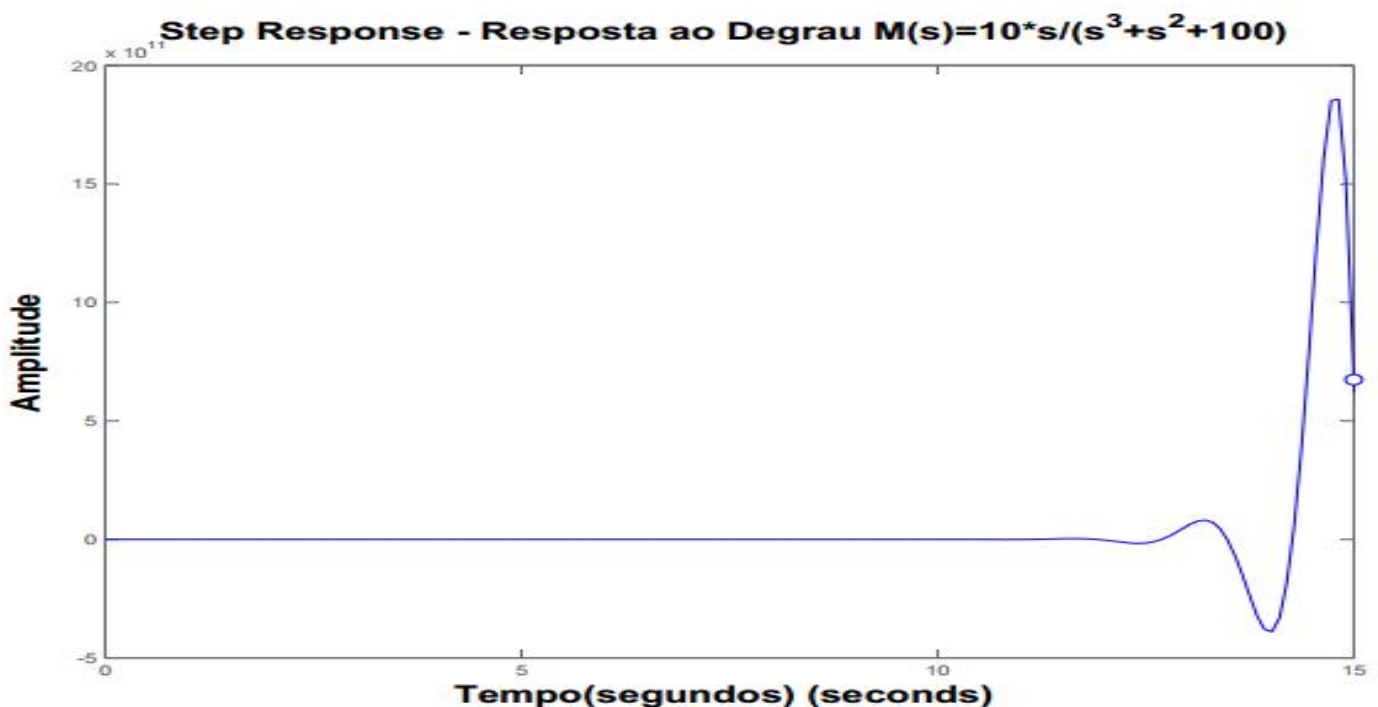
$$\frac{Y(s)}{D(s)} = \frac{10 \cdot s}{s^3 + s^2 + 100} \quad \begin{matrix} D(s) = 1/s \\ R(s) = 0 \end{matrix}$$

$$Ess = \lim_{s \rightarrow 0} \frac{-s \cdot G(s)}{1 + G(s) \cdot C(s)} \cdot D(s) \quad R(s) = 0$$

$$Ess = \lim_{s \rightarrow 0} \frac{-s \cdot 10 \cdot s}{s^3 + s^2 + 100} \cdot \frac{1}{s}$$

$$Ess = 0$$

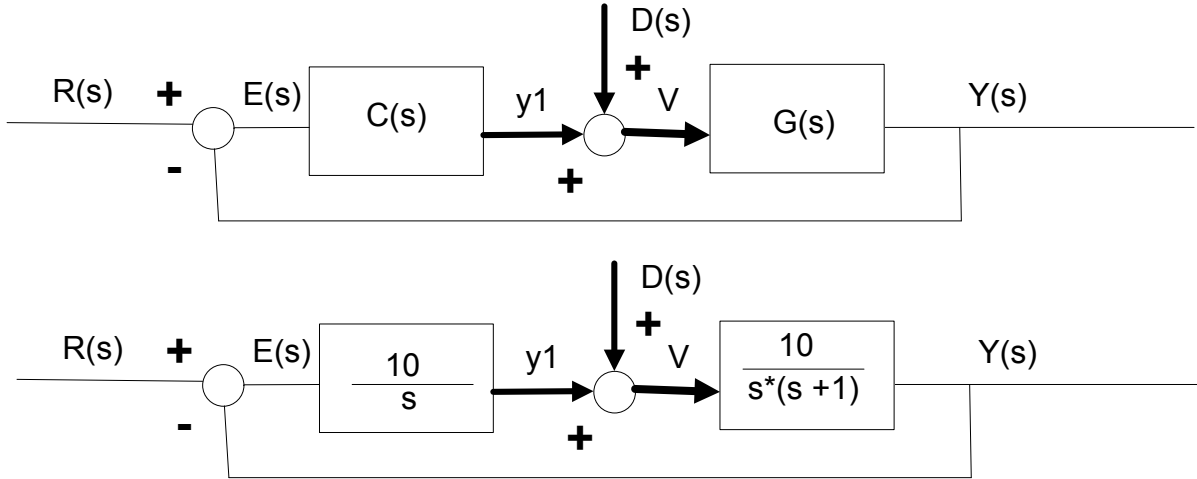
Análise do erro inválida pois Função de transferência tem pólos no semi-plano direito, resposta instável



Item(c) Refazer para $C(s) = 10/s$

$$Y(s) = \frac{C(s)*G(s)}{1 + G(s)*C(s)} *R(s) + \frac{G(s)}{1 + G(s)*C(s)} *D(s)$$

c) $r(t)$ = rampa unitária e $d(t)=0$



$$\frac{Y(s)}{R(s)} = \frac{C(s)*G(s)}{1 + G(s)*C(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{100}{s^3 + s^2 + 100} \quad \begin{matrix} D(s) = 0 \\ R(s) = 1/s^2 \end{matrix}$$

$$Ess = \lim_{s \rightarrow 0} \frac{s * R(s)}{1 + G(s)*C(s)} \quad D(s) = 0$$

$$Ess = \lim_{s \rightarrow 0} \frac{s}{[1 + G(s)*C(s)] * s^2} \quad D(s) = 0$$

$$Ess = \lim_{s \rightarrow 0} \frac{1}{\lim_{s \rightarrow 0} [1 + G(s)*C(s)] * s} = \frac{1}{A}$$

$$A = \lim_{s \rightarrow 0} [1 + G(s)*C(s)]$$

$$A = \lim_{s \rightarrow 0} \left[1 + \frac{100}{s^2 * (s+1)} * s \right] = 11$$

$$\lim_{s \rightarrow 0} \frac{10}{s*(s+1)} \quad A = \infty \quad \boxed{Ess = 0}$$

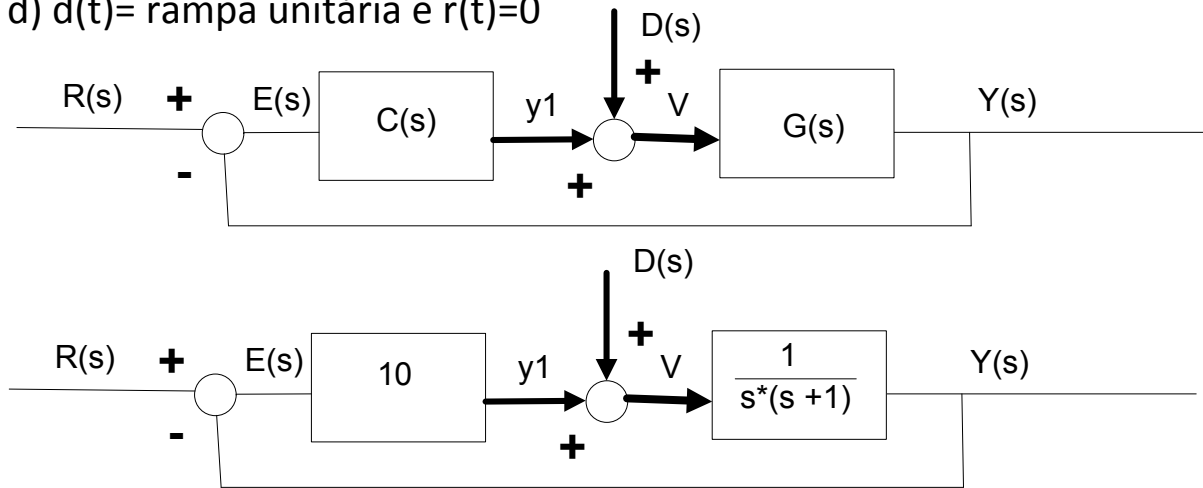
Análise do erro inválida pois Função de transferência tem pólos no semi-plano direito, resposta instável



Item(d) Refazer para $C(s) = 10/s$

$$Y(s) = \frac{C(s) \cdot G(s)}{1 + G(s) \cdot C(s)} \cdot R(s) + \frac{G(s)}{1 + G(s) \cdot C(s)} \cdot D(s)$$

d) $d(t) =$ rampa unitária e $r(t) = 0$



$$\frac{Y(s)}{D(s)} = \frac{G(s)}{1 + G(s) \cdot C(s)}$$

$$\frac{Y(s)}{D(s)} = \frac{10 \cdot s}{s^3 + s^2 + 100} \quad \begin{matrix} D(s) = 1/s^2 \\ R(s) = 0 \end{matrix}$$

$$Ess = \lim_{s \rightarrow 0} \frac{-s \cdot G(s)}{1 + G(s) \cdot C(s)} \cdot D(s) \quad R(s) = 0$$

$$Ess = \lim_{s \rightarrow 0} \frac{-s \cdot 10 \cdot s}{s^3 + s^2 + 100} \cdot \frac{1}{s^2}$$

$$Ess = -0,1$$

Análise do erro inválida pois Função de transferência tem pólos no semi-plano direito, resposta instável

